

MATHEMATICAL MODELLING FOR PLUG-FLOW SPRAY DRYER

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ABSTRACT The present work has focused on mathematical modelling for the spray drying operations using plug-flow conditions in order to compare two simple modelling approaches: the Characteristic Drying Rate Curve (CDRC) model and the Reaction Engineering Approach (REA) model. The comparison of modelling approaches is concerned with much practical importance. Mathematical models were formulated to predict particle's physical and biochemical characteristics. The formulated models were solved using a numerical method for the case of drying of a 'mono-disperse' stream of identical droplets. The predictions made by the CDRC and REA models were compared using the Excel programmes, which were used as process calculation tools. The REA model was found to be more realistic and accurate in predicting the particle's characteristics.

1. INTRODUCTION

The modelling plays an important role in prediction of the final product quality and in design and optimisation of the entire process and therefore the mathematical modelling becomes the most essential aspect of the drying technology, especially for the industrial spray drying operations and equipments. The drying process profoundly alters the physical, chemical and microbial properties of the food and in this way the quality of the final product [1]. Spray dryers have been used for nearly a century now, but still it is very difficult to model the performance of this type of process equipment, especially with respect to the quality of the product.

A first step is to recognise that the quality of the end product is the result of what a droplet or particle experiences (with respect to the temperature and humidity) during travelling in the spray dryer and how it responds. Therefore, it is extremely important to combine various processing parameters and environmental conditions with the factors that address the final product quality and this way the modelling comes in the picture. No exact correlations are available for mathematical modelling of the spray drying operation on the basis of only theoretical conditions. This is due to the unsteady state behaviour of heat and mass transfer to and from the droplet and the complex hydrodynamic actions prevailing in the process [1]. Moreover, there are complicated relations between original feed properties, drying process parameters and final product characteristics. Therefore, in the literature, the modelling for spray drying is still based on various drying and product parameters, empirical correlations, trial-and-error procedures and somehow on practical experience.

There have been many attempts to formulate a mathematical description for drying of single droplets since the 1970s. It was shown that the behaviour of industrial spray drying operation is well represented by the single droplets drying process. Researchers have introduced several

drying kinetics approaches based on various assumptions to characterise the droplet drying process. For instance, Sano and Keey [2], Meerdink and Riet [1] adopted the approach based on effective diffusivity concept. Another approach used by many researchers was based on the formation of crust with a receding droplet-bulk interface. Langrish and Kockel [3] used the approach (CDRC model) based on a characteristic drying rate curve with the assumption that at each volume-averaged free moisture content, there is a corresponding specific drying rate relative to the unhindered drying rate in the first drying period that is independent of the external drying conditions. This has been shown to be an effective approach. Chen and Lin [4] considered the approach based on a reaction engineering theory (REA model), assuming drying is a competitive process between ‘evaporation reaction’ and ‘condensation reaction’. However, it is important to note that each approach published in the literature has its own advantages and drawbacks.

The CDRC model is a more well-known approach and has been extensively used to simulate the industrial spray dryers. The REA model has been developed recently and is found to be useful in prediction of the behaviour of single droplets during drying. In this research work, these two simple and effective drying kinetics models were compared and discussed to examine the feasibility of these models in predicting droplet behaviour. The comparison of modelling approaches will be useful to have an accurate model for characterising the droplet drying process since a better approach can lead to better accuracy in predicting the product quality and designing the spray dryer. Models, which are too complex in nature or overly simplified, were not considered in this study. Here, simple mathematical models were formulated in order to predict the overall quality (physical and biological aspects) of the final product and to compare two drying kinetics models.

The case of drying of a ‘mono-disperse’ stream of uniform-sized droplets, which is the drying of a single stream of identical droplets with no horizontal velocity component, was studied for the comparison of two models. This allows the deeper understanding of the drying of free-falling droplets with enhanced simplicity, as one does not need to consider other phenomena such as droplet size distribution, droplet trajectories and entrainment effects. Furthermore, the drying of a mono-disperse stream of identical droplets requires one-dimensional analysis during the simulation, which makes the modelling fast, cheaper and effective. The device, which is commonly used in desktop ink-jet printers to dispense ink, generally known as the ‘ink-jet device’, was proposed in order to produce a single stream of identical droplets. This device can produce nearly uniform-sized droplets with identical characteristics under controlled conditions (the variation in droplet volume was observed less than $\pm 1\%$ [5], therefore the variation in diameter is even smaller). This idea of producing identical droplets using an ink-jet device was originally proposed by Prof. X. D. Chen from the University of Auckland in 2001.

2. MODEL OF THE PROCESS

An attempt has been made to model the co-current spray drying operation with plug flow conditions, which simplifies the prediction of the overall quality of the dried particles and helps in understanding the comparison of two modelling approaches. Few assumptions were introduced during modelling to make the analysis and simulation clearer. For plug-flow spray dryer, it is reasonable to model the process with an axial steady-state condition that assumes the droplet and drying medium properties vary only along the axial direction. The other assumptions were that the ink-jet device produces uniform-sized droplets and their shape remains completely spherical

during their entire travelling in the spray dryer. The final assumption was that the motion of droplets is not impeded by the motion of other droplets (free settling conditions).

Skim milk was used as a process fluid since the correlations for thermophysical properties of skim milk are easily available from the literature. The skim milk droplet was considered as a binary system made of water and skim milk solids. Mathematical models were formulated with the aid of mass, heat and momentum balance around the droplets and the drying medium phases. The detailed description of formulation of heat, mass and momentum transfer models was presented in the work by Patel [6] along with the particle's density and size models. The simulation of these models resulted in temperature, moisture content and velocity profiles respectively together with particle density and size. The drying rate was predicted using both the CDRC and REA models. These drying kinetics profiles govern the basis for prediction of both the physical and biological product quality. The biological characteristics of the dried particles were measured in terms of the residual microbial activity. In literature, most researchers have assumed steady-state conditions for the simulation of inactivation kinetics models; however, it is well known that the 'real' droplet drying is unsteady-state operation. The present study has used a new inactivation kinetics model [7] that considers the unsteady-state behaviour of the drying process in order to match the modelling conditions as close as possible to the industrial spray drying conditions. In addition, the drying kinetics and inactivation kinetics were coupled together during simulation to predict the overall quality of the final product. The attractive benefits of this concept are that it predicts both physical and biological properties simultaneously and provides better understanding about the dependency of inactivation kinetics on drying parameters. The overall quality of the end product was then compared using the CDRC and REA models.

3. DESCRIPTION OF THE EXCEL PROGRAMMES

The Excel programmes were developed for individual modelling approaches (CDRC and REA) as process calculation tools. The programming using Microsoft Excel makes the simulation more clear and understandable. Moreover, it is easy to see any changes at any point of the spray drying process and that is the most important aspect of the programmes. It is also straightforward to calculate any spray drying process knowing only a few inlet conditions using these simulation programmes. Each programme was divided in four parts, where specific calculations were performed using a small time interval. The first part of the programme introduces various inputs of the process and all inlet conditions and it defines major constants involved in the calculations. The second part of the programme calculates all necessary thermophysical, transport and equilibrium properties of vapour, water, dry air and skim milk including heat capacity, density, viscosity, thermal conductivity, latent heat of vaporisation, partial pressure, saturated vapour pressure, diffusion coefficient and drag coefficient. These properties aid in understanding of droplets drying process and needed as input to the models.

The third part of the programme represents the drying kinetics that shows basic mass, heat and momentum balance around both the droplet and drying medium phases. Water activity of the droplet, relative humidity of the drying medium, heat of wetting, heat transfer coefficient, mass transfer coefficient, drying rate and the distance travelled by the droplets at each time step were calculated along with the temperature, moisture content and velocity of both droplets and drying gas phases. Two individual programmes calculate all these parameters using two different modelling approaches. Furthermore, this part of the programme predicts the physical properties

of the final product, which includes particle density, moisture content and size. The fourth part of the programme deals with the inactivation kinetics for skim milk as a bioactive processing fluid. The fourth part estimates the inactivation rate constant and predicts the residual activity of the bioactive substance as a biological quality parameter. The formulated Excel programme simulates the drying kinetics and the inactivation kinetics together and provides better understanding of the relations between them. In addition, the programming in Microsoft Excel makes the comparison of the CDRC and REA models much easier.

4. SIMULATION

The dryer-wide simulation was carried out for a plug-flow spray dryer at atmospheric processing pressures. The solution of governing differential equations is one of the main tasks in the simulation of spray drying operations. The analytical solution of such equations is impossible due to the non-linear nature of the differential equations. For such a case, numerical methods offer an efficient and powerful means for simulation of mathematical models. In the current work, the first-order explicit forward finite difference method was employed to solve all the differential equations simultaneously since this method eliminates the solution of complex matrix systems, which one has to solve in backward schemes. The drying kinetics models were solved along with the particle density, particle size and inactivation kinetics models. Hot air was used as a drying medium for the evaporation of moisture from the skim milk droplets. The simulation required eight boundary conditions, which are listed in Table 1, in order to predict the physical and biological properties of the dried particles.

Parameters	Value
Droplet Diameter	0.0002 (m)
Droplet Temperature	60 (°C)
Droplet Moisture Content	1 (kg water/kg solids)
Mass Flow Rate of Skim Milk	10000 (droplets/second)
Air Temperature	200 (°C)
Air Humidity	0.01 (kg water/kg dry air)
Mass Flow Rate of Air	0.00075 (kg/s)
Bacterial Concentration	10^{20} per droplet

Table 1: The Inlet Conditions for Simulation using Two Modelling Approaches

5. RESULTS AND DISCUSSION

The CDRC and REA models were examined using a set of boundary conditions listed in Table 1. The models were also tested for varying initial air temperature, air flow rate, droplet diameter, droplet moisture content and air humidity conditions [4, 6]. The simulation predicted the approximately the same values for particle's physical characteristics (density and size) for the final product using both the models with all sets of drying conditions [4, 6]. Despite of this, there

was a significant difference in the residual microbial activity profiles, which were obtained using the inactivation kinetics of *Lactobacillus Acidophilus* during drying of skim milk droplets. Figure 1 shows that the residual activity of *Lactobacillus Acidophilus* with the REA model was found to be of a greater magnitude. This is due to the prediction of lower drying rates with the REA model over the CDRC model, which further resulted in the system having a lower rate of change of particle's moisture content. This can be observed using the rate of change of moisture content profile in Figure 1. According to the inactivation kinetics model, the inactivation rate was directly proportional to the rate of moisture content change (dX/dt). It is noticeable from Figure 1 that the initial dX/dt with the REA model was very small in comparison with the CDRC model. The lower rate with the REA model resulted in a lower inactivation near the inlet region of a spray dryer leading to a greater residual activity of bacteria at the exit. A greater magnitude of the exit residual activity indicates a better biochemical quality of the final product. This shows the importance of having a rather accurate drying model during modelling of drying processes.

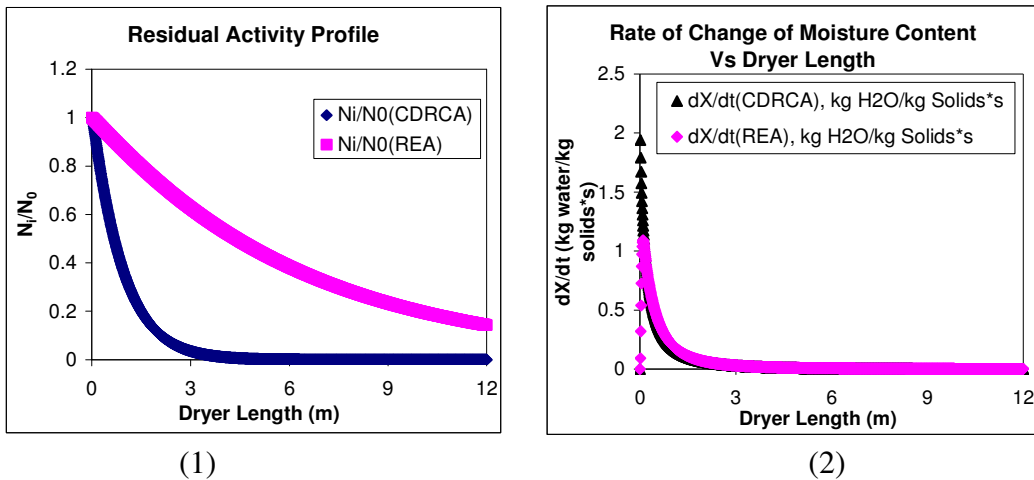


Figure 1: Comparison of Two Modeling Approaches using (1) Residual Microbial Activity Profile and (2) Rate of Change of Moisture Content Profile

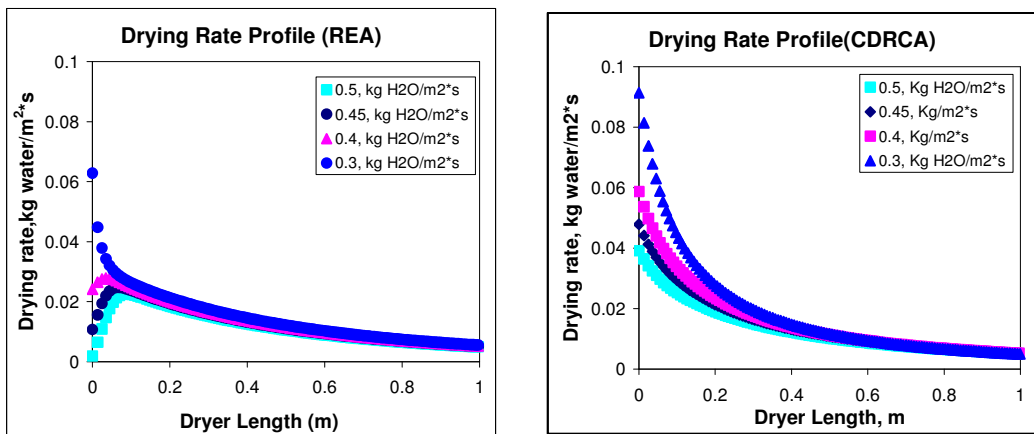


Figure 2: Comparison of Drying Rate Profiles using the Droplets having Different Initial Solids Content Values of 50, 45, 40 and 30 % (By Weight).

Figure 2 shows the drying rate profiles using two different modelling approaches with different values of initial total solids content. It can be seen that the simulation programmes predicted higher initial drying rates with the CDRC model over the REA model. Moreover, the CDRC model predicted a falling-rate drying period at all feed concentration, while the REA model predicted a small increasing-rate drying period for the droplets having 40, 45 and 50 wt % feed concentration. The drying rate profiles with the REA model seem more accurate. The reason for being that is the CDRC model initially assumes that the droplet surface is unsaturated from the beginning of the drying process leading to the system having moisture content equal to the critical moisture content. This would definitely predict a drying process having a falling-rate drying period from the beginning of the process at any value of initial solids content. The REA model, in fact, doesn't have any similar assumption and has been shown to correlate the experimental values better [8]. Hence, the REA model was found to be more accurate since it is based on experimental data rather than any assumptions.

6. CONCLUSIONS

Simple models were developed and solved using the simulation programmes made in Microsoft Excel in order to compare two modelling approaches. The CDRC and REA were compared in terms of the prediction of product quality and drying parameters. It was shown that if the quality change is dependent on the rate of water loss, two models has predicted quite different outcomes. Both models have estimated approximately the same values for particle's physical characteristics [4, 6, 8]. The REA model predicted a better final quality for the all sets of drying conditions tested [4, 6, 8]. Furthermore, the REA model does not neglect a drying period for droplets having a high water content during modelling and hence it is applicable to even feed with much lower initial solids concentrations. The CDRC model is still good due to its simplicity and it is attractive to the modelling of drying processes where the simplicity is more essential.

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